HEF-003-1501002 Seat No. ________

M. Phil. (Mathematics) (Sem. I) (CBCS) Examination

December – 2017

Combinatorics & Graph Theory : CMT - 10002

Faculty Code : 003

Subject Code : 1501002

Time : **3** Hours] [Total Marks : **100**

Instruction : (1) Attempt all the questions.

(2) Each question carries equal marks.

(3) There are five questions.

(4) Figures to the right indicate full marks.

1 Attempt the following : **20**

(a) Define : Degree of a vertex

Also state and prove first theorem of graph theory.

Is it true that the number of even vertices in a graph is always odd? Justify your answer.

(b) Define : Regular graph and Bipartite graph

Give an example of a graph which is regular and bipartite. Also prove that a graph is bipartite if and only if it has no odd cycle.

OR

(b) Define : Adjacency matrix and incidence matrix of a graph and state any two properties of each. Also draw the Petersen graph *P(5,2)* and obtain adjacency and incidence matrices for the same.

2 Attempt the following : **20**

(a) Define: Bridge, Tree and Cycle.

Also prove that : An edge of a graph is a bridge if and only if it is not a cycle edge.

(b) Define Spanning tree and vertex connectivity.

Also prove that: A graph G is connected if and only if it has a spanning tree.

3 Attempt the following : **20**

(a) Prove that: Any graph G is a tree if and only if there is precisely one path between any two vertices of G.

(b) Prove that the following are equivalent for a graph G with n vertices

(i) G is a tree

(ii) G is an acyclic graph with $(n - 1)$ edges

(iii) G is a connected graph with $(n - 1)$ edges.

4 Attempt the following : **20**

(a) Define : Eulerian graph

Also prove that a connected graph is Eulerian if and only if degree of every vertex is even.

OR

(a) Give an example of a graph which is regular, complete, Eulerian and bipartite.

Also prove that a connected graph G has an Euler trail if and only if it has at least two vertices of odd degree.

(b) Define : Hamiltonian graph

Also prove that : If G is a simple graph with n vertices with $n \geq 3$ and for every ν of G, $d(\text{v}) \geq \frac{n}{2}$ then G is Hamiltonian.

5 Attempt any four : **20**

(a) Define with examples :

(i) Dominating set

(ii) Minimal dominating set

Also prove that : Every connected graph G is order n has a dominating set S whose complement $V - S$ is also a dominating set.

(b) Define : Isolate and enclave. Also give an example of an enclave less dominating set.

Also prove that : A dominating set S is a minimal dominating set if and only if for each vertex, $u \in S$ one of the following two conditions holds :

(i) u is an isolate of S

(ii) There exists a vertex $v \in V - S$ for which $N(v) \bigcap S = \{u\}$.

(c) State sum rule and answer the following question :

How many two digit numbers can be found which are divisible by 2 and its first digit is odd ?

(d) State product rule and answer the following question :

Using alphabets how many m letters acronyms can be formed without using the letters x, y and z.

(e) State only symmetry and addition properties of binomial co-efficients. Also state pigeonhole principle in simple form and strong form.

Also answer the following question :

A bouquet of flowers is being prepared using Rose, Sunflower and Mogara in such a way that, either at least 10 Roses or at least 9 Sunflowers or at least 5 Mogaras.